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The tangential velocity excess of the Milky Way satellites

Marius Cautun[★] and Carlos S. Frenk

Department of Physics, Institute for Computational Cosmology, Durham University, South Road Durham DH1 3LE, UK

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ABSTRACT

We estimate the systemic orbital kinematics of the Milky Way classical satellites and compare them with predictions from the Λ cold dark matter (Λ CDM) model derived from a semi-analytical galaxy formation model applied to high-resolution cosmological N -body simulations. We find that the Galactic satellite system is atypical of Λ CDM systems. The subset of 10 Galactic satellites with proper motion measurements has a velocity anisotropy, $\beta = -2.2 \pm 0.4$, which lies in the 2.9 per cent tail of the Λ CDM distribution. Individually, the Milky Way satellites have radial velocities that are lower than expected for their proper motions, with 9 out of the 10 having at most 20 per cent of their orbital kinetic energy invested in radial motion. Such extreme values are expected in only 1.5 per cent of Λ CDM satellites systems. In the standard cosmological model, this tangential motion excess is unrelated to the existence of a Galactic ‘disc of satellites’. We present theoretical predictions for larger satellite samples that may become available as more proper motion measurements are obtained.

Key words: Galaxy: halo – Local Group – cosmology: theory – dark matter.

1 INTRODUCTION

Several predictions of the current cosmological paradigm – the Λ cold dark matter (Λ CDM) model – agree with observations such as those of the temperature anisotropies of the cosmic microwave background radiation and galaxy clustering (e.g. see Frenk & White 2012). None the less, the model has been claimed to be in disagreement with some properties of the Local Group satellites. These claims include the observations that there are far fewer dwarf galaxies than there are dark matter substructures (Klypin et al. 1999; Moore et al. 1999, a discrepancy misleadingly dubbed the ‘missing satellites’ problem); that the internal structure of the most massive subhaloes is incompatible with that of known satellite galaxies (the ‘too-big-to-fail’ problem; Boylan-Kolchin, Bullock & Kaplinghat 2011); and that a large fraction of satellites seem to rotate in a thin plane (the ‘planes of satellites’ problem; Kroupa, Theis & Boily 2005; Ibata et al. 2013; Pawlowski & Kroupa 2013). The first two ‘problems’ can be resolved by including realistic galaxy formation models (e.g. Sawala et al. 2016), but the latter is more challenging. Systematic studies of the Milky Way (MW) and M31 planes of satellites show that such configurations are uncommon, with only ~ 10 per cent of Λ CDM galactic-mass systems having more prominent planes than those in the Local Group (Cautun et al. 2015a,b).

In this Letter, we compare the kinematics of the Galactic satellites with the predictions of Λ CDM. We do so for the subset of 10 satellites that have *Hubble Space Telescope* (HST) proper motions

(the 11 classical ones except Sextans). Previous studies have focused on two aspects of satellite kinematics: measuring the clustering of the orbital poles and reconstructing satellite orbits. The orbital poles are more clustered than an isotropic distribution, with the clustering being the largest for a subset of 8 of the 11 classical satellites (Pawlowski & Kroupa 2013). Orbit reconstruction is more challenging since the outcome is sensitive to both the mass and the radial density profile of the MW halo (e.g. Lux, Read & Lake 2010; Barber et al. 2014), both of which are poorly constrained (Wang et al. 2015, and references therein). This leads to large uncertainties in the recovered orbits, and thus, a comparison with theoretical predictions is not very informative. To overcome such limitations, in this study, we compare the velocity anisotropy parameter and the fraction of kinetic energy in radial motion between observations and theory. These two quantities are largely insensitive to the mass of the halo and to its radial density profile.

2 DATA AND SIMULATIONS

Our observational sample consists of the 10 bright Galactic satellites that have HST proper motions. These objects and the sources of their proper motion measurements are Sagittarius – Pryor, Piatek & Olszewski (2010), Large Magellanic Cloud (LMC) and Small Magellanic Cloud (SMC) – Kallivayalil et al. (2013), Draco – Pryor, Piatek & Olszewski (2015), Ursa Minor – Piatek et al. (2005), Sculptor – Piatek et al. (2006), Carina – Piatek et al. (2003), Fornax – Piatek et al. (2007), Leo II – Piatek, Pryor & Olszewski (2016) and Leo I – Sohn et al. (2013). We used satellite distances and heliocentric velocity values from the McConnachie (2012) compilation. To obtain the radial and tangential velocity components with respect to

[★] E-mail: m.c.cautun@durham.ac.uk

the Galactic Centre, we followed the procedure described in Cautun et al. (2015b). We generate 1000 Monte Carlo realizations of the MW system in which we sample the satellite positions and proper motions from Gaussian distributions centred on the most likely values of each quantity and with dispersion equal to the uncertainties. These are transformed from heliocentric to Galactic coordinates, with the Monte Carlo realizations used to compute confidence intervals. The largest uncertainty is in the tangential velocities, with 1σ errors varying from 20 to 55 km s⁻¹ (median value 40 km s⁻¹).

The theoretical model is based on the semi-analytic galaxy formation model of Henriques et al. (2015) applied to the Millennium II Λ CDM dark matter cosmological simulation (Boylan-Kolchin et al. 2009), which has been rescaled to correspond to the Planck-1 values of the cosmological parameters (for details, see Henriques et al. 2015). Our sample consists of haloes in the mass range, $M_{200} = (0.8\text{--}3.0) \times 10^{12} M_{\odot}$, where M_{200} is the mass enclosed by a spherical overdensity of 200 times the critical density. Our results are insensitive to the host halo mass, so we use a broad mass range motivated by the large uncertainties in the MW halo mass (Wang et al. 2015) and the advantages of having a large sample of MW analogues. We find 3672 such host haloes. We restrict the satellite selection to galaxies with a minimum stellar mass of $10^5 M_{\odot}$ found within a distance of 300 kpc from the central galaxy. For each host, we select the 10 satellites with the largest stellar mass. In the case of the MW observations, we have proper motions for 10 satellites out of 12 objects brighter than $M_V = -8.6$ (the classical satellites and Canes Venatici). To check for systematic biases, we constructed a second satellite catalogue by randomly selecting 10 out of the 12 objects with the largest stellar mass. We found that the two catalogues have the same satellite velocity distribution, so, for simplicity, we limit our analysis to the 10 brightest satellites.

We construct mock satellite catalogues to account for the uncertainties in the radial and tangential velocity components. We start by ranking the satellites according to their distance from the central galaxy. We do the same for the MW satellites. Then, the simulated satellites are assigned the errors corresponding to the MW satellite with the same rank, for example, the innermost satellite in the simulation is linked to the MW innermost one. To model observational uncertainties, for every satellite, we add to each velocity component a random value generated from a Gaussian distribution centred on zero with dispersion equal to the error reported for that velocity component. We repeat this procedure 10 times for each host, resulting in 36 720 MW mocks.

3 RESULTS

The velocity anisotropy parameter, β , provides a simple measure of the kinematical properties of satellite galaxies. It is defined as

$$\beta = 1 - \frac{\sum_i V_{\text{tan};i}^2}{2 \sum_i V_{\text{rad};i}^2}, \quad (1)$$

where $V_{\text{rad};i}$ and $V_{\text{tan};i}$ denote the radial and tangential velocity components of satellite i with respect to the central galaxy. The sum is over all the satellites associated with a host halo, which, in our case, is 10. The β parameter takes values in the range of $-\infty$ to 1, with $\beta < 0$, $\beta = 0$ and $\beta > 0$ describing circularly biased, isotropic and radially biased orbits, respectively.

Fig. 1 shows the distribution of β values for the 10 brightest satellites of galactic-mass haloes in our sample. We show the distribution for mock satellite catalogues and also for the original cosmological simulation (i.e. in the absence of velocity errors). In both cases, the satellite systems have radially biased orbits, with a most likely

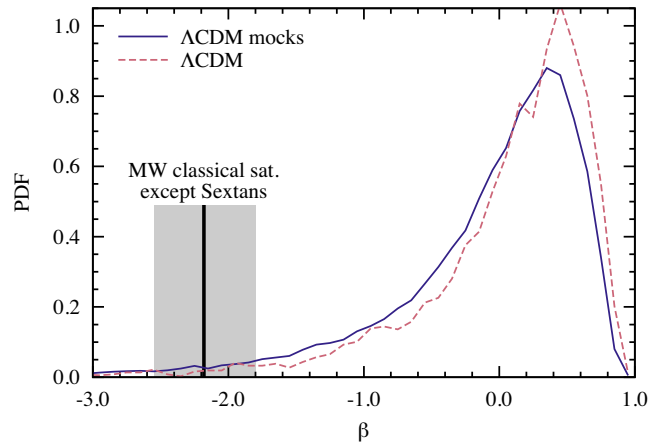


Figure 1. The distribution of the velocity anisotropy, β , for the 10 brightest satellites of MW-mass haloes. We show results for the cosmological simulation (dashed line) and for mock satellite catalogues that account for observational uncertainties (solid line). The vertical line shows the measured value, $\beta = -2.2 \pm 0.4$, for the MW satellites, and the grey shaded region shows the 1σ uncertainty interval. Only 2.9 per cent of mock systems have a lower value of β than the MW system.

value, $\beta \simeq 0.4$, but the β distribution in the mock catalogues is slightly shifted towards lower values. The shift is due to the transverse velocity errors being an order of magnitude larger than the radial velocity errors. On average, this leads to an overestimation of $V_{\text{tan};i}^2$ by a larger amount than of $V_{\text{rad};i}^2$, and thus a systematic reduction in β .

The Galactic satellites have $\beta = -2.2 \pm 0.4$, which means that they have tangentially biased motions. This agrees with previous studies that, using fewer Galactic satellites with *HST* proper motions, also found a preference for tangential motions (e.g. Watkins, Evans & An 2010; Pawlowski & Kroupa 2013). The β value of the Galactic satellites, marked with a vertical line in Fig. 1, lies in the tail of the theoretical prediction, with only 2.9 per cent of Λ CDM mock catalogues having an even more extreme value.

Fig. 2 shows the distribution of tangential versus radial motion for individual satellites. We characterize this by the fraction of kinetic energy, $f_{\text{E};\text{rad}} = \frac{V_{\text{rad}}^2}{V^2}$, along the radial direction. A satellite that, at a given moment, has a preferentially tangential motion corresponds to $f_{\text{E};\text{rad}} < \frac{1}{3}$, while a satellite that has a preferentially radial motion corresponds to $f_{\text{E};\text{rad}} > \frac{1}{3}$. Λ CDM predicts that at any moment, 49 per cent of the satellites have $f_{\text{E};\text{rad}} < \frac{1}{3}$, which increases to 52 per cent for the Galactic mock satellite catalogues.

The distribution of $f_{\text{E};\text{rad}}$ values for the Galactic satellites is dominated by tangential motions, with $f_{\text{E};\text{rad}} < 0.2$ for 9 out of the 10 satellites (thick solid line in Fig. 2); on average, Λ CDM predicts only four such objects. To quantify the significance of the disagreement between observations and theory, we cannot just compute the fraction of mock catalogues that have nine or more satellites with $f_{\text{E};\text{rad}} = 0.2$, since this would be an *a-posteriori-defined* test that disregards the *look elsewhere* effect. This problem can be overcome by performing an extended Kolmogorov–Smirnov test that accounts for additional sources of scatter beyond just those due to Poisson statistics. We define the maximum difference between the cumulative distribution functions (CDF) of the data, CDF_{data} , and of the mean for the mock catalogues, CDF_{mean} , as

$$D = \max_{f_{\text{E};\text{rad}}} |\text{CDF}_{\text{data}}(f_{\text{E};\text{rad}}) - \text{CDF}_{\text{mean}}(f_{\text{E};\text{rad}})|. \quad (2)$$

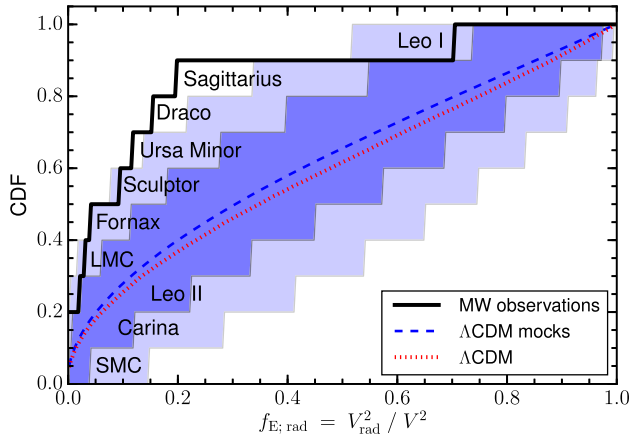


Figure 2. The distribution of the kinetic energy fraction in radial motion, $f_{E,rad} = \frac{V_{rad}^2}{V^2}$, for the 10 brightest satellites. The dashed line shows the median trend for the Λ CDM Galactic mocks. The darker and lighter shaded regions show the 1σ and 2σ scatter regions. The distribution of MW satellites, which is shown by the solid line, is consistent with the mocks at the 1.5 per cent level. We also show the median expectation in the absence of observational errors (dotted line).

The Galactic satellite system has $D_{MW} = 0.5$. For each mock catalogue, we compute D given by equation (2) with the CDF of the data replaced by the CDF of the $f_{E,rad}$ values in that particular mock catalogue. The probability of obtaining a deviation as extreme as that observed in the data is given by the fraction of mock catalogues with D values larger than D_{MW} . Only 1.5 per cent of mock catalogues show a larger deviation than the data.

4 DISCUSSION

The 10 MW satellites with measured proper motions have tangentially biased motions to an extent rarely found in Λ CDM. Only 2.9 per cent of Λ CDM systems have lower values of β than the MW satellite system. Even fewer, 1.5 per cent, show deviations in the CDF of $f_{E,rad}$ that are as extreme as those measured for the MW. The two discrepancies are expressions of the same property, as may be seen in Fig. 3. Selecting the 5 per cent of mock catalogues with the lowest values of β results in a distribution that is biased towards low $f_{E,rad}$ values, similarly to that measured in the real data. In the following, we consider possible reasons behind the disparity between observations and theory. We focus on the test illustrated in Fig. 2, i.e. the CDF of $f_{E,rad}$, since that test shows the largest discrepancy and thus is the most constraining.

In Fig. 3, we investigate whether the preference for tangential motions is somehow related to the Galactic ‘disc of satellites’ (Lynden-Bell 1976; Kroupa et al. 2005; Libeskind et al. 2005). Very few haloes have satellite systems similar to that in the MW (Pawlowski et al. 2014), i.e. that are as thin and have highly clustered orbital poles, so, to have good statistics, we need to study each of these two aspects separately. First, we select the 5 per cent of mock satellite catalogues that have the thinnest planes of satellites. These are the systems with the smallest values of c/a , where a and c are, respectively, the major and minor axes of the inertia tensor of the satellite distribution. This subsample of haloes, shown with a thin solid line in Fig. 3, has the same CDF of $f_{E,rad}$ values as the overall sample. Thus, the flattening of a satellite distribution is uncorrelated with its degree of tangential motion.

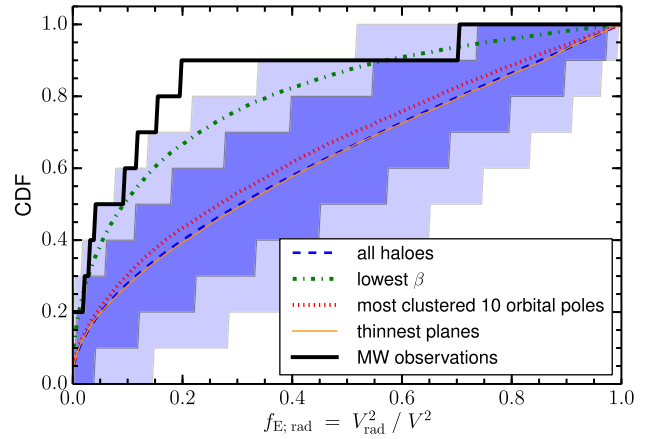


Figure 3. As Fig. 2, but showing the median expectation for all Λ CDM haloes (dashed line) and for subsets consisting of the 5 per cent of haloes that have the lowest β values (dash-dotted line), the most clustered satellite orbital poles (dotted line) and the thinnest satellite planes (solid thin line). The plot shows that low β values are highly correlated with low $f_{E,rad}$ values and thus both are expressions of the same phenomenon. The presence of a satellite plane or of coherent rotation has little effect on the $f_{E,rad}$ values and thus the two effects are largely independent.

Of the 10 Galactic satellites with measured proper motions, 7 have orbital poles that are significantly clustered on the sky (Pawlowski & Kroupa 2013). For each of our mock satellite systems, we identify the set of n satellites (out of 10) that have the most strongly clustered orbital poles, i.e. the smallest angular dispersion in the direction of the orbital poles (see equation 6 in Cautun et al. 2015b). For each value of n , we then select the 5 per cent of haloes that have the most clustered orbital poles. These subsamples show a small preference for tangential motions compared to the full sample, with the excess being the largest for $n = 10$. The dotted line in Fig. 3 shows the most extreme case, $n = 10$. The MW data are consistent with this subsample at the 3.3 per cent level. The weak correlation between preferentially tangential motions, and clustering of the orbital poles may not be very relevant for the Galactic satellites. When considering the clustering of all satellites (i.e. $n = 10$), the MW is in the 15th, not the 5th, percentile of the distribution; mock catalogues corresponding to that percentile behave exactly like the full sample. The Galactic satellites are extreme for $n = 7$, but the shift in the CDF in that case is much smaller than for the $n = 10$ case shown in Fig. 3. Thus, a strong clustering of the orbital poles of satellites is, at most, only weakly associated with an excess of tangential motion.

The LMC and the SMC are thought to have been accreted as a pair (e.g. Besla et al. 2012), so the two galaxies could have correlated orbital dynamics. This is unlikely to explain the tangential velocity excess of the Galactic satellites since group accretion is common in Λ CDM: when studying the 11 brightest satellites, Wang, Frenk & Cooper (2013) found accretion of satellite groups with two or more members in five out of their six haloes. None the less, we tested for the effect of group accretion by excluding the SMC from the sample. We repeated the analysis for systems of nine satellites and found only a small reduction in the difference between data and theory: the nine MW satellites lie in the 3.0 per cent tail of the distribution for Λ CDM.

Satellite proper motions are difficult to measure and could potentially be affected by unknown systematic errors. To reduce the Galactic tangential velocity excess to a 1σ disagreement, the proper motion of each satellite would have to be overestimated by

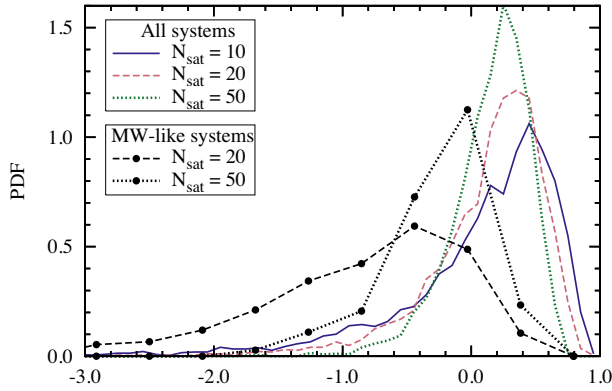


Figure 4. The distribution of velocity anisotropy, β , for the 10 (solid line), 20 (dashed line) and 50 (dotted line) brightest satellites of Λ CDM galactic-mass haloes. The lines with symbols show the same distribution but for haloes selected to resemble the MW, which are the 5 per cent of haloes whose 10 brightest satellites have the lowest values of β . The results do not include velocity errors.

45 per cent. Recently, Casetti-Dinescu & Girard (2016) published a new ground-based proper motion measurement for Draco that is $\sim 6\sigma$ discrepant from the Pryor et al. *HST* measurement. The ground-based measurement gives a much lower tangential velocity, $(90 \pm 16) \text{ km s}^{-1}$, compared to the *HST* value, $(210 \pm 25) \text{ km s}^{-1}$. Taking this value would ease the discrepancy between theory and observations, with only 8 out of the 10 Galactic satellites having $f_{\text{E,rad}} \leq 0.2$, which would make the MW system a 9 per cent outlier. It remains to be determined by future observations which of the two Draco proper motion measurements is correct and whether the *HST* measurements are affected by as yet unknown systematic errors. Two other concerns might be the limited resolution and the absence of baryonic effects in the cosmological simulation used here. We checked for these possibilities by analysing the COCO simulation (Hellwing et al. 2016, which has 100 times better mass resolution) and the APOSTLE Local Group simulations (Sawala et al. 2016, which include realistic baryonic physics). We found good agreement between the results of these simulations and those of the one used in this study.

The fraction of kinetic energy invested in radial motions, $f_{\text{E,rad}}$, depends on the position of the satellite along its orbit, being the smallest at pericentre and apocentre. The low $f_{\text{E,rad}}$ value found for the Galactic satellites could be interpreted as implying that 9 of the 10 satellites are close to either pericentre or apocentre to a larger extent than is normally found in Λ CDM. The $f_{\text{E,rad}}$ value also depends on the orbital ellipticity, being smaller for circularly biased orbits. Thus, the discrepancy between data and theory could alternatively indicate that the Galactic satellites have orbits that are, on average, closer to circular than is typical in Λ CDM. This would mean that MW halo mass estimates based on satellite orbits (e.g. Barber et al. 2014) are biased low.

More observations are required to decide which, if any, of the above explanations is correct, or, alternatively, if the excess of tangential motions is an indication of new physics in the dark sector. There are two main directions in which this analysis can be extended: measuring proper motions for fainter Galactic satellites or performing similar tests for external galaxies. Figs 4 and 5 show theoretical predictions for the expected behaviour of β and $f_{\text{E,rad}}$ as proper motion measurements become available for a larger number of satellites. The *Gaia* mission will reduce the uncertainties in the proper motions of several of the

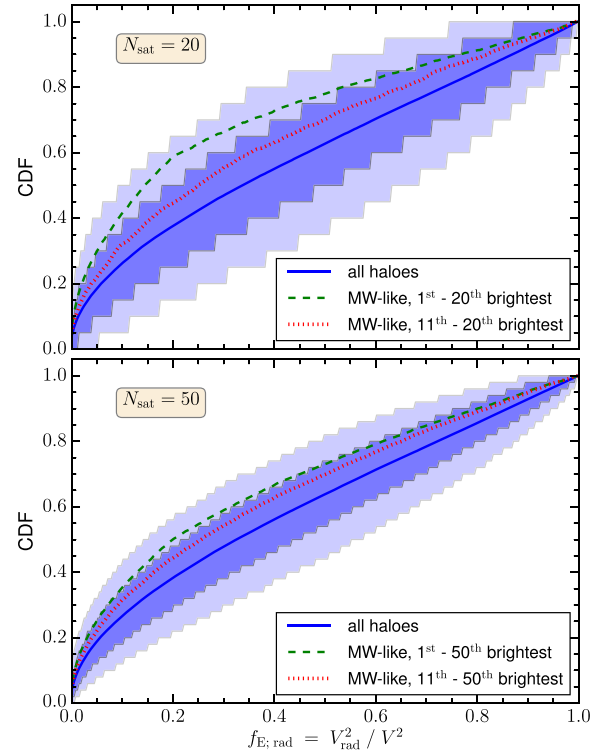


Figure 5. The distribution of the fraction of kinetic energy in radial motions, $f_{\text{E,rad}}$, for the 20 (top panel) and 50 (bottom panel) brightest satellites of Λ CDM galactic-mass haloes. The darker and lighter shaded regions show the 1σ and 2σ scatter regions. The dashed line shows the expectation for haloes selected to resemble the MW, which are haloes for which at least 7 of the 10 brightest satellites have $f_{\text{E,rad}} \leq 0.2$ (which corresponds to ~ 5 per cent of the population). The dotted line shows the expectation for the MW-like systems when excluding the 10 brightest satellites, i.e. when considering only the 11th–20th or the 11th–50th brightest samples. The results do not include velocity errors.

classical satellites and should obtain new measurements for fainter objects, especially for those within $\sim 100 \text{ kpc}$ from the Sun (Wilkinson & Evans 1999). The proper motions of more distant Galactic satellites and of those in M31 could be measured by a dedicated multiyear *HST* programme and by a follow-up with *James Webb Space Telescope* and *Wide-Field Infrared Survey Telescope* (Kallivayalil et al. 2015).

Fig. 4 shows that the velocity anisotropy, β , decreases as fainter satellites are included in the sample, with the typical value varying from $\beta = 0.45$ for the 10 brightest satellites to $\beta = 0.25$ for the 50 brightest satellites. The distribution of β becomes more peaked and narrower for larger satellite samples. To make predictions constrained by the already existing data for the MW, where the 10 brightest satellites have very low β values, we select the 5 per cent of haloes whose 10 brightest satellites have the lowest velocity anisotropy ($\beta \leq -1.3$). The velocity anisotropy of the 20 or 50 brightest satellites remains biased low for these systems relative to the full sample of haloes.

In Fig. 5, we show the CDF of the kinetic energy fraction in radial motion for the 20 and 50 brightest satellites. While the median trend hardly changes, the scatter is much reduced as the number of satellites increases. The dashed curves show the expected behaviour for Λ CDM systems chosen to be similar to the MW, that is haloes for which at least 7 of the 10 brightest satellites have $f_{\text{E,rad}} \leq 0.2$. These MW-like systems show systematically larger tangential motions

even when excluding the 10 brightest satellites that were used in the first place to select the sample.

5 SUMMARY

We have found that the bright satellites of the MW have larger tangential orbital motions than expected from Λ CDM cosmological simulations. This excess is most clearly manifest in the fraction of kinetic energy along the radial direction, $f_{E,\text{rad}}$, with 9 of the 10 MW satellites with *HST* proper motion measurements having $f_{E,\text{rad}} < 0.2$. Such extreme values are found in at most 1.5 per cent of Λ CDM galactic satellite systems. This conclusion, of course, relies on the accuracy of current *HST* proper motion measurements, which has been called into question by a recent measurement of Draco using ground-based data. In Λ CDM, the tangential motion excess is unrelated to the existence of a Galactic ‘disc of satellites’ and cannot be explained by the accretion of satellite groups. More satellites with measured proper motions are required to check if the observed excess is merely an indication that the MW is atypical or if it poses a problem for the Λ CDM model.

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